STABILITY OF A GAS BOUNDARY LAYER ON A HEATED SURFACE WITH A WEAK NEGATIVE PRESSURE GRADIENT

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It is well known that cooling of a surface leads to stability of a laminar gas boundary layer and retardation of the transition to the turbulent flow regime, while heating reduces stability and accelerates the transition. The former property is regarded as an effective means of laminarizing a surface, for example, the wing surface of an aircraft [1, 2].

These properties have been established both experimentally and theoretically for constant surface temperature and absence of a pressure gradient. It was noted relatively recently that in the presence of a surface temperature gradient the absolute opposite effects can be observed – destabilization of a laminar gas boundary layer upon surface cooling and stabilization upon heating [3-5].

The stability of laminar flows and their transition to turbulence on curvilinear surfaces under nonisothermal conditions had not been studied previously, with the exception of [6], which carried out experiments on an NASA 0012 wing profile oriented at zero and small attack angles in an aerodynamic tube. It was established that heating of the nose of the profile, which comprised about 10% of its total length, produced an increase in stability with the transition point shifting down the flow by approximately 20%.

The present authors have carried out a theoretical study of the effect of significant negative pressure gradients on stability of a laminar boundary layer on a uniformly heated surface. A significant increase in stability and decrease in growth increments was obtained, which agrees qualitatively with the experimental data of [6].

The goal of the present study is to demonstrate that even for a very slight pressure gradient flow stability increases on a surface uniformly heated with sufficient intensity. As will become evident below, the case of weak pressure gradients differs qualitatively from that studied previously by us in several new features. Such a study is of practical importance from the viewpoint of experimental verification of the stabilization effect, since, for example, by placing a plate at a slight angle of attack in an aerodynamic tube the high Reynolds number values necessary for fixing the beginning of transition and the transition itself can be obtained.

We will consider a planar infrasonic laminar gas boundary layer on a heated surface with negative pressure gradient. The mathematical model is a system of equations consisting of the continuity equation, the Navier–Stokes equation, written for the two projections of the velocity vector, and the energy equation neglecting viscous dissipation:

$$\begin{gathered}
\frac{\partial\rho}{\partial t}+\frac{\partial\rho u}{\partial x}+\frac{\partial\rho v}{\partial y}=0,\\
\rho\frac{Du}{Dt}=-\frac{\partial p}{\partial x}+\frac{2}{3}\frac{\partial}{\partial x}\left[\mu\left(2\frac{\partial u}{\partial x}-\frac{\partial v}{\partial y}\right)\right]+\frac{\partial}{\partial y}\left[\mu\left(\frac{\partial u}{\partial y}+\frac{\partial v}{\partial x}\right)\right],\\
\rho\frac{Dv}{Dt}=-\frac{\partial p}{\partial y}+\frac{\partial}{\partial x}\left[\mu\left(\frac{\partial u}{\partial y}+\frac{\partial v}{\partial x}\right)\right]+\frac{2}{3}\frac{\partial}{\partial y}\left[\mu\left(2\frac{\partial v}{\partial y}-\frac{\partial u}{\partial x}\right)\right],\\
\rho C_p\left(u\frac{\partial T}{\partial x}+v\frac{\partial T}{\partial y}\right)=\frac{\partial}{\partial x}\left(\lambda\frac{\partial T}{\partial y}\right)+\frac{\partial}{\partial y}\left(\lambda\frac{\partial T}{\partial y}\right),\\
\frac{D}{Dt}=\frac{\partial}{\partial t}+u\frac{\partial}{\partial x}+v\frac{\partial}{\partial y}.
\end{gathered}\tag{1}$$

The boundary conditions are:

$$u=0,\ T=T_w,\ v=0\ (y=0),$$
$$u\to u_e,\ T\to T_e\ (y\to\infty).$$

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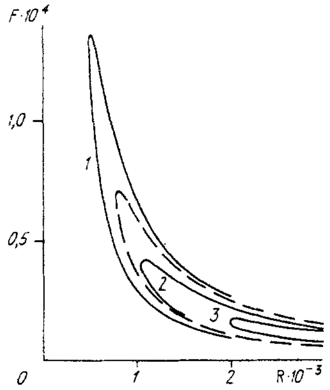

Fig. 1

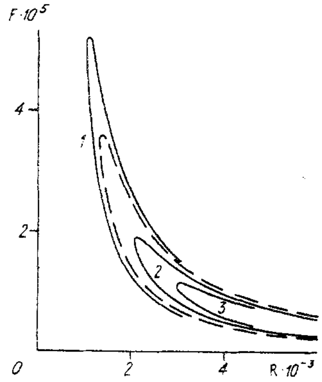

Fig. 2

Here x and y are the longitudinal and transverse coordinates; u and v are the corresponding velocity components, p is pressure, ρ is gas density; C_p is the gas heat capacity at constant pressure, T is the absolute temperature; μ and λ are the viscosity and thermal conductivity coefficients.

Linearizing system (1), we obtain equations describing the development of small disturbances in the boundary layer. Using the obvious assumption of smallness of the wavelength of the disturbance (Tollmien–Schlichting wave) as compared to the characteristic length for change in temperature, which is comparable to the size of the surface L, assuming a plano-parallel boundary layer [1, 2] we represent the flow function perturbation in the form of a plane wave $\varphi(y)\exp(i\alpha(x - ct))$ [$\varphi(y)$ is the flow function amplitude, $\alpha = 2\pi/\lambda_0$ is the wave number, c is the wave phase velocity]. By eliminating the pressure perturbation from the linearized equations the system obtained can be written in the form of one equation which is quite cumbersome, and therefore will not be presented. In the variables

$$\xi = x/L,\ \eta = \left(\frac{u_e}{x\nu_e}\right)^{1/2}\int_0^y \frac{\rho}{\rho_e}\,dy$$

it was presented in [3, 5].

Thus, the problem under consideration has been reduced to determination of eigenvalues of a linear boundary problem with homogeneous boundary conditions. For its solution we will require coefficients which contain the velocity, temperature, and viscosity distributions and their derivatives over the thickness of the boundary layer. These can be found by solving the boundary layer temperature equations, which in the variables used here have the self-similar form

$$(kf'')' + \frac{n+1}{2}ff'' + n(\psi - f'^2) = \Phi_1,\ \left(\frac{k}{\Pr}\psi'\right)' + \frac{n+1}{2}f\psi' = \Phi_2, \tag{2}$$

where $\Phi_1 = \xi\left(f'\frac{\partial f'}{\partial\xi} - f''\frac{\partial f}{\partial\xi}\right)$; $\Phi_2 = \xi\left(f'\frac{\partial\psi}{\partial\xi} - \psi'\frac{\partial f}{\partial\xi}\right)$; Pr is the Prandtl number; and the prime denotes differentiation with respect to the variable η.

The boundary conditions are:

$$\begin{gathered} f = 0,\ f' = 0,\ \psi = t_w(\xi)\ (\eta = 0), \\ f' = 1,\ \psi = 1\ (\eta = \infty). \end{gathered} \tag{3}$$

For the Folkner–Sken type boundary layers considered below, where $u_e = u_0x^n$, the boundary problem of Eqs. (2), (3) takes on a self-similar form ($\Phi_1 = 0$, $\Phi_2 = 0$). The eigenvalues of the linear boundary problem were determined numerically by a computer using a modified orthogonalization method, with numerical coefficients determined by numerical solution of the problem of Eqs. (2) and (3) by the Keller method [3].

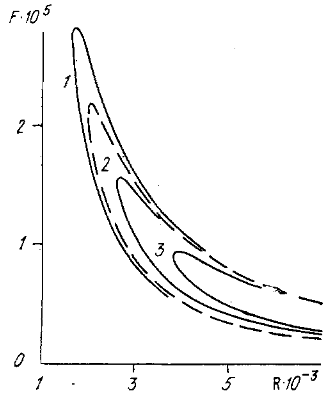

Fig. 3

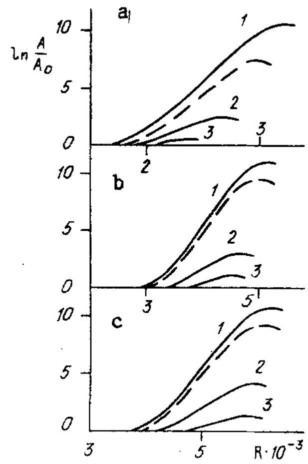

Fig. 4

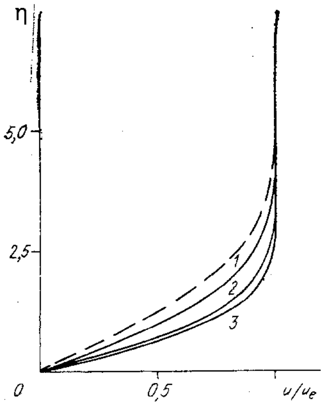

Fig. 5

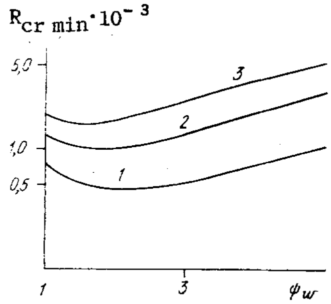

Fig. 6

The calculation results are shown in Figs. 1-6. Figures 1-3 (with pressure gradient parameter n = 0.04, 0.07, 0.1, respectively) show neutral stability curves in the plane of dimensionless frequency $F = \omega\nu_e/u_e^2$ and Reynolds number $R = (u_e x/\nu_e)^{1/2}$ for a temperature factor $\psi_w = T_w/T_e$ = 2, 3, 4 (lines 1-3). Here, as well as in Figs. 4 and 5 the dashed line corresponds to an isothermal boundary layer. In all the figures at relatively small values of ψ_w we initially see a decrease in the minimum critical Reynolds number $R_{cr\ min}$ and expansion of the unstable frequency range, after which the Reynolds number increases and the frequency range narrows with further growth in ψ_w. Thus, for n = 0.07 at $\psi_w = 2$, $R_{cr\,min}$ decreases only insignificantly, while at $\psi_w = 4$ it increases by approximately 2.4 times, which corresponds to an increase in the laminar overflow segment by almost six times.

Even more interesting conclusions can be drawn from the curves shown in Fig. 4 of integral perturbation increase increments $\ln\frac{A}{A_0} = -\int\limits_{x_0}^{x} \alpha_i dx$ (where A is the perturbation amplitude, A_0 is the initial amplitude, x_0 is the value of the x coordinate corresponding to the left branch of the neutral stability curve for a given F, α_i is the local increment, i.e., the imaginary component of the wave number α), where a corresponds to n = 0.04, $F = 1.4\cdot10^{-5}$; b) n = 0.07, $F = 7.3\cdot10^{-6}$; c) n = 0.1, $F = 6\cdot10^{-6}$; with curves 1-3 being ψ_w values as in Figs. 1-3.

It is evident that at relatively small ψ_w perturbations increase more intensely than under isothermal conditions. On the other hand, for large values of ψ_w of the perturbation increments can be significantly less than the isothermal values. Thus, for n = 0.1 and ψ_w = 3 a decrease of somewhat less than two times takes place in maximum perturbation increase, while for ψ_w = 4 the decrease is almost seven times. This corresponds to a decrease in maximum perturbation amplitude by nine times in the first case and three orders of magnitude in the second.

A complete explanation of this nonmonotonic action of nonisothermal conditions on stability characteristics is quite complicated. However the increase in stability at high ψ_w is apparently related to the significantly higher filling of the velocity profile in the wall region as compared to isothermal conditions. This can be seen from the calculation results presented in Fig. 5 for n = 0.07. The curve enumeration corresponds to the ψ_w values of Figs. 1-3.

The character of the temperature factor's effect on stability noted above is shown in Fig. 6 by curves of $R_{cr\,min}$ as a function of ψ_w for n = 0.04, 0.07, 0.1 (lines 1-3); these are nonmonotonic and have a minimum, the position of which depends on the pressure gradient parameter. Thus, while for n = 0.1 the minimum is at $\psi_w = 1.5$, for n = 0.04 it occurs at $\psi_w = 2$. For further decrease the position shifts into the region of higher ψ values. The fact that no experiments were performed under significantly nonisothermal conditions is apparently the reason that stabilization due to uniform surface heating was not observed previously.

Thus, even a slight negative temperature gradient exerts a significant effect upon stability characteristics on the heated surface. For sufficiently intense heating boundary stability layer may be significantly greater than under isothermal conditions.

LITERATURE CITED

1. H. Shlichting, Boundary Layer Theory, 7th edn., McGraw-Hill, New York (1979).
2. Yu. S. Kachanov, V. V. Kozlov, and V. Ya. Levchenko, Development of Turbulence in a Boundary Layer [in Russian], Nauka, Novosibirsk (1982).
3. V. V. Struminskii, Yu. B. Lebedev, and V. M. Fomichev, "Effect of a temperature gradient along a surface upon the extent of a laminar boundary layer," Dokl. Akad. Nauk SSSR, 299, No. 4 (1986).
4. Yu. B. Lebedev and V. M. Fomichev, "Effect of the extent of the heated portion of the surface on stability characteristics of a laminar gas boundary layer," Izv. Sib. Otd. Akad. Nauk SSSR, Ser. Tekh. Nauk, No. 21(6) (1988).
5. V. V. Struminskii, A. V. Dovgal', Yu. B. Lebedev, et al., "Control of perturbation development in a boundary layer by nonuniform heating of the surface," Izv. Sib. Otd. Akad. Nauk SSSR, Ser. Tekh. Nauk, No. 11(3) (1988).
6. D. B. Landrum and J. M. Macha, "Influence of a heated leading edge on boundary layer growth, stability, and transition," AIAA Paper No. 1259, New York (1987).